

Multiscale probability transformation of basic probability assignment

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Abstract

Decision making is still an open issue in the application of Dempster-Shafer evidence theory. A lot of works have been presented for it. In the transferable belief model (TBM), pignistic probabilities based on the basic probability assignments are used for decision making. In this paper, multiscale probability transformation of basic probability assignment based on the belief function and the plausibility function is proposed, which is a generalization of the pignistic probability transformation. In the multiscale probability function, a factor q based on the Tsallis entropy is used to make the multiscale probabilities diversified. An example is shown that the multiscale probability transformation is more reasonable in the decision making.

Keywords: Decision making, Dempster-Shafer evidence theory, Transferable belief model, Pignistic probability transformation, Multiscale

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1. Introduction

Since first proposed by Dempster [1], and then developed by Shafer [2], the Dempster-Shafer theory of evidence, which is also called Dempster-Shafer theory or evidence theory, has been paid much attentions for a long time and continually attracted growing interests. Even as a theory of reasoning under the uncertain environment, Dempster-Shafer theory has an advantage of directly expressing the “uncertainty ”by assigning the probability to the subsets of the set composed of multiple objects, rather than to each of the individual objects, so it has been widely used in many fields [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

Due to improve the Dempster-Shafer theory of evidence, many studies have been devoted for combination rule of evidence [15, 16, 10, 17, 18], confliction problem [19, 20, 21, 22, 23], generation of mass function [24, 25, 26, 27, 28], uncertain measure of evidence [29, 30, 31, 32], and so on [33, 34, 35, 36, 37, 38, 39]. One open issue of evidence theory is the decision making based on the basic probability assignments, many works have been done to construct a reasonable model for the decision making [40, 41, 42, 43, 44, 45].

In the transferable belief model (TBM) [40], pignistic probabilities are used for decision making. The transferable belief model is presented to represent quantified beliefs based on belief functions. TBM was constructed by two levels. The credal level where beliefs are entertained and quantified by belief functions. The pignistic level where beliefs can be used to

make decisions and are quantified by probability functions. The main idea of the pignistic probability transformation is to transform the multi-elements subsets into singleton subsets by an average method. Though the pignistic probability transformation is widely used, it can not describe the unknown for the multi-elements subsets. Hence, a generalization of the pignistic probability transformation called multiscale probability transformation of basic probability assignment is proposed in this paper, which is based on the belief function and the plausibility function. The proposed function can be calculated with the difference between the belief function and the plausibility function, we call it multiscale probability function and denote it as a function $MulP$. In the multiscale probability function, a factor q based on the Tsallis entropy [46] is used to make the multiscale probabilities diversified. When the value of q equals to 0, the proposed multiscale probability transformation can be degenerated as the pignistic probability transformation.

The rest of this paper is organized as follows. Section 2 introduces some basic Preliminaries about the Dempster-Shafer theory and the pignistic probability transformation. In section 3 the multiscale probability transformation is presented. Section 4 uses an example to illustrate the effectiveness of the multiscale probability transformation. Conclusion is given in Section 5.

2. Preliminaries

2.1. Dempster-Shafer theory of evidence

Dempster-Shafer theory of evidence [1, 2], also called Dempster-Shafer theory or evidence theory, is used to deal with uncertain information. As

an effective theory of evidential reasoning, Dempster-Shafer theory has an advantage of directly expressing various uncertainties. This theory needs weaker conditions than bayesian theory of probability, so it is often regarded as an extension of the bayesian theory. For completeness of the explanation, a few basic concepts are introduced as follows.

Definition 1. Let Ω be a set of mutually exclusive and collectively exhaustive, indicted by

$$\Omega = \{E_1, E_2, \dots, E_i, \dots, E_N\} \quad (1)$$

The set Ω is called frame of discernment. The power set of Ω is indicated by 2^Ω , where

$$2^\Omega = \{\emptyset, \{E_1\}, \dots, \{E_N\}, \{E_1, E_2\}, \dots, \{E_1, E_2, \dots, E_i\}, \dots, \Omega\} \quad (2)$$

If $A \in 2^\Omega$, A is called a proposition.

Definition 2. For a frame of discernment Ω , a mass function is a mapping m from 2^Ω to $[0, 1]$, formally defined by:

$$m : 2^\Omega \rightarrow [0, 1] \quad (3)$$

which satisfies the following condition:

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{A \in 2^\Omega} m(A) = 1 \quad (4)$$

In Dempster-Shafer theory, a mass function is also called a basic probability assignment (BPA). If $m(A) > 0$, A is called a focal element, the union of all focal elements is called the core of the mass function.

Definition 3. For a proposition $A \subseteq \Omega$, the belief function $Bel : 2^\Omega \rightarrow [0, 1]$ is defined as

$$Bel(A) = \sum_{B \subseteq A} m(B) \quad (5)$$

The plausibility function $Pl : 2^\Omega \rightarrow [0, 1]$ is defined as

$$Pl(A) = 1 - Bel(\bar{A}) = \sum_{B \cap A \neq \emptyset} m(B) \quad (6)$$

where $\bar{A} = \Omega - A$.

Obviously, $Pl(A) \geq Bel(A)$, these functions Bel and Pl are the lower limit function and upper limit function of proposition A , respectively.

2.2. Pignistic probability transformation

In the transferable belief model (TBM) [40], pignistic probabilities are used for decision making. The definition of the pignistic probability transformation is shown as follows.

Definition 4. Let m be a BPA on the frame of discernment Ω . Its associated pignistic probability function $BetP_m : \Omega \rightarrow [0, 1]$ is defined as:

$$BetP_m(\omega) = \sum_{A \subseteq P(\Omega), \omega \in A} \frac{1}{|A|} \frac{m(A)}{1 - m(\phi)}, \quad m(\phi) \neq 1 \quad (7)$$

where $|W|$ is the cardinality of subset A . The process of pignistic probability transformation (PPT) is that basic probability assignment transferred to probability distribution. Therefore, the pignistic betting distance can be easily obtained by PPT.

3. Multiscale probability transformation of basic probability assignment

In the transferable belief model (TBM) [40], pignistic probabilities are used for decision making. The transferable belief model is presented to represent quantified beliefs based on belief functions. The main idea of the pignistic probability transformation is to transform the multi-elements subsets into singleton subsets by an average method. Though the pignistic probability transformation is widely used, it is not reasonable in the Example 1.

Example 1. Suppose there is a frame of discernment of a, b, c , the BPA is given as follows.

$$m(\{a\}) = 0.2, m(\{b\}) = 0.7, m(\{b, c\}) = 0.05, m(\{a, b, c\}) = 0.05.$$

In the pignistic probability transformation, for $m(\{a, b, c\}) = 0.05$, the result will be $a = b = c = 0.05/3$. Actually it is not reasonable, $m(\{a, b, c\}) = 0.05$ means the sensor can not judge the target belongs to which classes, it represents a meaning of “unknown”. In other word, only according to $m(\{a, b, c\}) = 0.05$, nothing can be obtained except “unknown”. In this situation, average is used in the pignistic probability transformation, which is one of the methods to solve the problem. Compared with the average, weighted average is more reasonable in many situations. In this paper, the weighted average is represented by the difference between the belief function and the plausibility function, whose definition is shown as follows.

Definition 5. Let m be a BPA on the frame of discernment Ω . The differ-

ence function d_m is defined as:

$$d_m(\omega) = Pl(\omega) - Bel(\omega), \quad \omega \in \Omega \quad (8)$$

Definition 6. The weight is defined as:

$$Weight_m(\omega) = \frac{d_m(\omega)}{\sum_{\alpha \in A} d_m(\alpha)}, \quad \omega \in A, A \subseteq P(\Omega) \quad (9)$$

Based on the weighted average idea, a factor q , which is proposed in the Tsallis entropy [46], is used to highlight the weights. Thus, the definition of multiscale probability function $MulP$ is shown as follows.

Definition 7. Let m be a BPA on the frame of discernment Ω . Its associated multiscale probability function $MulP_m : \Omega \rightarrow [0, 1]$ on Ω is defined as:

$$MulP_m(\omega) = \sum_{A \subseteq P(\Omega), \omega \in A} \left(\frac{(Pl(\omega) - Bel(\omega))^q}{\sum_{\alpha \in A} (Pl(\alpha) - Bel(\alpha))^q} \frac{m(A)}{1 - m(\phi)} \right), \quad m(\phi) \neq 1 \quad (10)$$

where $|W|$ is the cardinality of subset A . q is a factor based on the Tsallis entropy to amend the proportion of the interval. The transformation between m and $MulP_m$ is called the multiscale probability transformation.

Actually, the part of the Eq. 10 $\frac{(Pl(\omega) - Bel(\omega))^q}{\sum_{\alpha \in W} (Pl(\alpha) - Bel(\alpha))^q}$ denotes the weight of element ω based on normalization, which is replaced the averaged $\frac{1}{|W|}$ in the pignistic probability function.

Theorem 3.1: Let m be a BPA on the frame of discernment Ω . Its associated multiscale probability $MulP_m$ on Ω is degenerated as the pignistic probability $BetP_m$ when q equals to 0.

Proof: When q equals to 0, $(Pl(\omega) - Bel(\omega))^q$ equals to 1, the multiscale probability function will be calculated as follows:

$$MulP_m(\omega) = \sum_{A \subseteq \Omega, \omega \in A} \left(\frac{1}{\sum_{\alpha \in A} 1} \cdot \frac{m^\Omega(W)}{(1 - m^\Omega(\phi))} \right), \forall \omega \in \Omega \quad (11)$$

Then, it can obtain:

$$MulP_m(\omega) = \sum_{A \subseteq \Omega, \omega \in A} \left(\frac{1}{|A|} \cdot \frac{m^\Omega(A)}{(1 - m^\Omega(\phi))} \right), \forall \omega \in \Omega \quad (12)$$

From Eq. 11 and Eq. 12, we can see that when the value of q equals to 0, the proposed multiscale probability function can be degenerated as the pignistic probability function.

Theorem 3.2: Let m be a BPA on the frame of discernment Ω . If the belief function equals to the plausibility function, its associated multiscale probability $MulP_m$ on is degenerated as the pignistic probability $BetP_m$.

Proof: Given a BPA m on the frame of discernment Ω , for each $\omega \in \Omega$, when the belief function equals to the plausibility function, namely $Bel(\omega) = Pl(\omega)$, the bel is a probability distribution P [40], then $MulP$ is equal to $BetP$.

For example, let Ω be a frame of discernment and $\Omega = \{a, b, c\}$, if it satisfies with $Bel(a) = Pl(a)$, $Bel(b) = Pl(b)$, $Bel(c) = Pl(c)$, the BPA on the frame must be satisfied with $m(a) + m(b) + m(c) = 1$. In this situation, the multiscale probability will be degenerated as the pignistic probability.

Corollary: If bel is a probability distribution P , then $MulP$ is equal to P .

Theorem 3.3: Let m be a BPA on the frame of discernment $\Omega = a, b, c$. If the differences between the belief function and the plausibility function is the same, the multiscale probability transformation can be degenerated as the pignistic probability transformation.

Proof: The same as the proof of theorem 3.1.

An illustrative example is given to show the calculation of the multiscale probability transformation step by step.

Example 2. Let Ω be a frame of discernment with 3 elements. We use a, b, c to denote element 1, element 2, and element 3 in the frame. One body of BPA is given as follows:

$$m(\{a\}) = 0.2,$$

$$m(\{b\}) = 0.3,$$

$$m(\{c\}) = 0.1,$$

$$m(\{a, b\}) = 0.1,$$

$$m(\{a, b, c\}) = 0.3.$$

Step 1 Based on Eq. 5 and Eq. 6, the values of the belief function and the plausibility function of elements a, b, c can be obtained as follows:

$$Bel(a) = 0.2, Pl(a) = 0.6,$$

$$Bel(b) = 0.3, Pl(b) = 0.7,$$

$$Bel(c) = 0.1, Pl(c) = 0.4.$$

Step 2 Calculate the difference between the belief function and the plausibility function:

$$d_m(a) = Pl(a) - Bel(a) = 0.6 - 0.2 = 0.4,$$

$$d_m(b) = Pl(b) - Bel(b) = 0.7 - 0.3 = 0.4,$$

$$d_m(c) = Pl(c) - Bel(c) = 0.4 - 0.1 = 0.3.$$

Step 3 Calculate the weight of each element in Ω . Assumed that the value of q equals to 1.

When $A = \{a, b\}$,

$$Weight_m(a) = \frac{(Pl(a) - Bel(a))}{\sum_{\alpha \in W} (Pl(\alpha) - Bel(\alpha))} = 0.4 / (0.4 + 0.4) = 0.5,$$

$$Weight_m(b) = \frac{(Pl(b) - Bel(b))}{\sum_{\alpha \in W} (Pl(\alpha) - Bel(\alpha))} = 0.4 / (0.4 + 0.4) = 0.5.$$

When $W = \{a, b, c\}$,

$$Weight_m(a) = \frac{(Pl(a) - Bel(a))}{\sum_{\alpha \in W} (Pl(\alpha) - Bel(\alpha))} = 0.4 / (0.4 + 0.4 + 0.3) = 0.364,$$

$$Weight_m(b) = \frac{(Pl(b) - Bel(b))}{\sum_{\alpha \in W} (Pl(\alpha) - Bel(\alpha))} = 0.4 / (0.4 + 0.4 + 0.3) = 0.364,$$

$$Weight_m(c) = \frac{(Pl(c) - Bel(c))}{\sum_{\alpha \in W} (Pl(\alpha) - Bel(\alpha))} = 0.3 / (0.4 + 0.4 + 0.3) = 0.272.$$

Step 4 The value of the multiscale probability function can be obtained based on above steps.

$$MulP_m(a) = 0.2 + 0.1 * 0.5 + 0.3 * 0.364 = 0.3592,$$

$$MulP_m(b) = 0.3 + 0.1 * 0.5 + 0.3 * 0.364 = 0.4592,$$

$$MulP_m(c) = 0.1 + 0.3 * 0.272 = 0.1816.$$

4. Case study

In this section, an illustrative example is given to show the effect of the multiscale probability function when the value of q changes.

Example 3. Let Ω be a frame of discernment with 3 elements, namely $\Omega = \{a, b, c\}$.

Given one body of BPAs:

$$m(\{a\}) = 0.3,$$

$$m(\{b\}) = 0.1,$$

$$m(\{a, b\}) = 0.1,$$

$$m(\{a, c\}) = 0.2,$$

$$m(\{a, b, c\}) = 0.3.$$

Based on the pignistic probability transformation, the results of the pignistic probability function is shown as follows:

$$BetP_m(a) = 0.55,$$

$$BetP_m(b) = 0.25,$$

$$BetP_m(c) = 0.20.$$

According to the proposed function in this paper, the results of the multiscale probability function can be obtained through the follow steps.

Firstly, the values of belief function and the plausibility function can be obtained as follows:

$$Bel(a) = 0.3, Pl(a) = 0.9,$$

$$Bel(b) = 0.1, Pl(b) = 0.5,$$

$$Bel(c) = 0, Pl(c) = 0.5.$$

Then, the differences between the belief functions and the plausibility functions can be calculated:

$$d_m(a) = Pl(a) - Bel(a) = 0.6 - 0.2 = 0.6,$$

$$d_m(b) = Pl(b) - Bel(b) = 0.7 - 0.3 = 0.4,$$

$$d_m(c) = Pl(c) - Bel(c) = 0.4 - 0.1 = 0.5.$$

Based on the definition of the multiscale probability transformation, the values of $MulP_m$ can be obtained. There are 20 cases where the values of q starting from Case 1 with $q = 0$ and ending with Case 11 when $q = 10$ as shown in Table 1. The values of $MulP_m$ for these 20 cases is detailed in Table 1 and graphically illustrated in Fig. 1.

Table 1: The values of multiscale probability function when the values of q changes.

Cases	$MulP(a)$	$MulP(b)$	$MulP(c)$
q=0	0.5500	0.2500	0.2000
q=1	0.5891	0.2200	0.1909
q=2	0.6275	0.1931	0.1794
q=3	0.6638	0.1703	0.1659
q=4	0.5970	0.1518	0.1512
q=5	0.7267	0.1374	0.1360
q=6	0.7526	0.1266	0.1208
q=7	0.7751	0.1187	0.1062
q=8	0.7944	0.1130	0.0926
q=9	0.8109	0.1089	0.0801
q=10	0.8250	0.1061	0.0689

According to the Table 1 and the Fig. 1, on one hand, when the value of q increased, the probability of the element which has larger weight is increased,

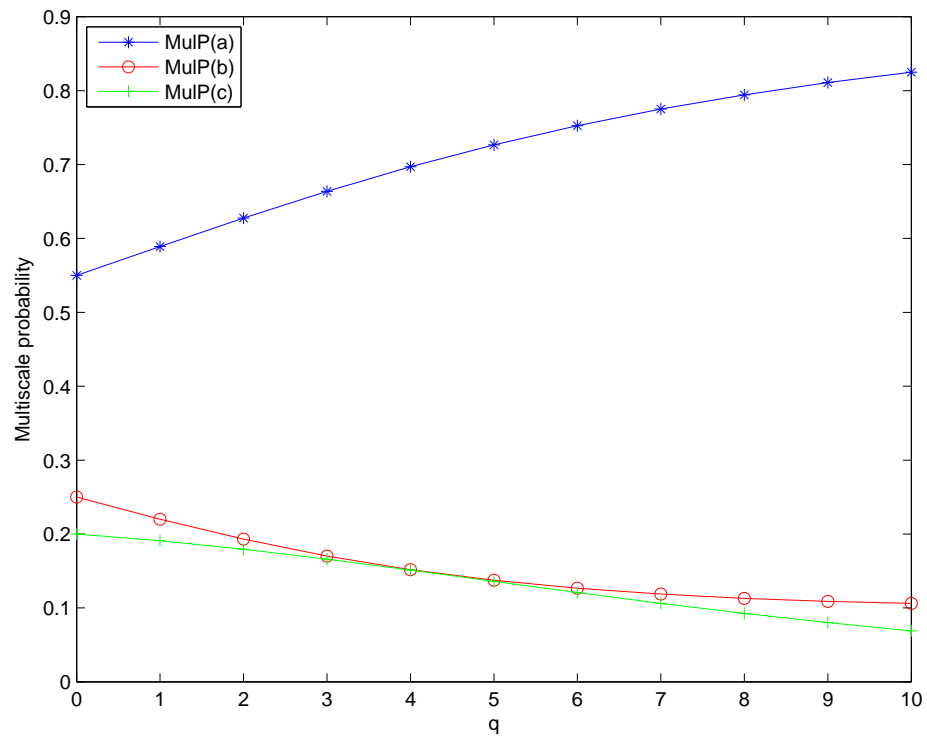


Figure 1: The values of multiscale probability function when the values of q changes.

and the probability of the element which has smaller weight is decreased. For example, the element a starting with probability 0.5500, and ending with probability 0.8250. The element b starting with probability 0.2500, and ending with probability 0.1061.

On the other hand, according to the Table 1, the option ranking of the the values of $MulP_m$ can be obtained. It is starting with $MulP_m(a) \succ MulP_m(b) \succ MulP_m(c)$, and ending with $MulP_m(a) \succ MulP_m(c) \succ MulP_m(b)$. It is mainly because $MulP_m$ is impact of the values of q . This principle makes the multiscale probability function has the ability to highlight the proportion of each element in the frame of discernment.

Note that when the value of q equals to 0, the values of pignistic probability $BetP_m$ is the same as the values of multiscale probability $MulP_m$, which is proposed in this paper. In other word, the multiscale probability function is a generalization of the pignistic probability function.

5. Conclusion

In the transferable belief model(TBM), pignistic probabilities are used for decision making. In this paper, a multiscale probability transformation of basic probability assignment based on the belief function and the plausibility function, which is a generalization of the pignistic probability transformation is proposed. In the multiscale probability function, a factor q is proposed to make the multiscale probability function has the ability to highlight the proportion of each element in the frame of discernment. When the value of q equals to 0, the multiscale probability transformation can be degenerated as

the pignistic probability transformation. An illustrative case is provided to demonstrate the effectiveness of the multiscale probability transformation.

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Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this article.

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